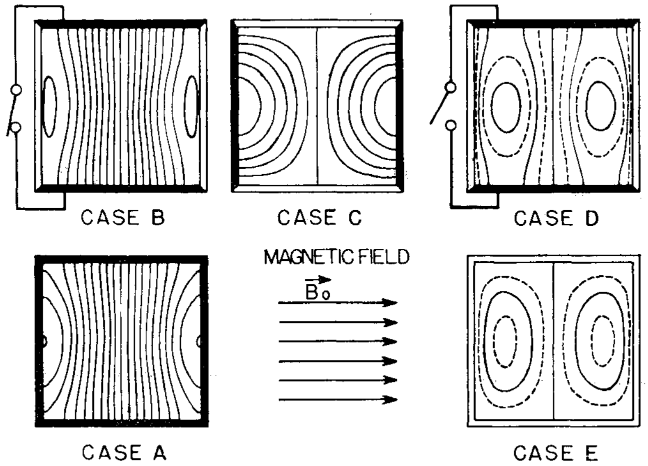

Fig. 1 Patterns of electric current for constant flow rate in square ducts with conducting (black) or nonconducting (white) walls. Hartmann number $M = B_0a(\sigma/\eta)^{1/2} = 2$, where B_0, a, σ, and η denote flux density of external magnetic field, half the length of a side of duct, conductivity, and viscosity of fluid, respectively. The horizontal walls are short-circuited in case B but unconnected in case D

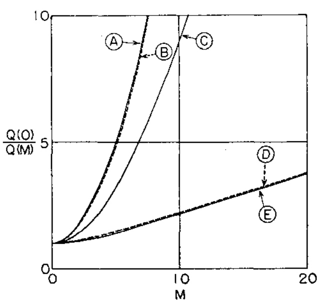

Fig. 2 Reciprocal of rate of flow vs Hartmann number. $Q(M)$ denotes the volume rate of flow at Hartmann number M, and $Q(0)$ that at $M = 0$; (A) Chang and Lundgen,[1] (B) Tani,[2] (C) Lundgen, Atabeck, and Chang,[4] (D) Tani,[2] and (E) Shercliff[3]

As a matter of fact, there is a strong resemblance between cases A and B both in current pattern and flow rate (Figs. 1 and 2). Indeed, only 21 and 16% of the current (this percentage tending to zero as M or the height/breadth ratio of the duct becomes infinitely large) comes from the vertical walls in case A at Hartmann number $M = 5$ and 10, respectively. (Thus Hartmann flow with conducting walls[1] is exactly the same as with nonconducting walls with net current.)

Although the current patterns in cases D and E are different in the neighborhood of the horizontal walls, the flow rates are expected to be only slightly different when $M^{1/2}$ is large, because the electric resistance to the current loop in the boundary layer on the horizontal wall of case E is of the order of $M^{1/2}$, whereas the principal contribution of the order of M comes from the boundary layer on the vertical wall in both cases D and E.

Finally, case C[4] has the same mechanism of flow resistance as cases A and B at large M, but the current loop in C suffers electric resistance of the order of $M^{1/2}$ in the boundary layer on the horizontal wall. Thus the flow rate for this case at large M may be expected to be similar to case A with the conductivity of the fluid somewhat decreased.

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Unification of Matrix Methods of Structural Analysis

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Nomenclature

S = column matrix of internal stresses
b_1 = transformation matrix of unknowns into internal stresses
b_0 = transformation matrix of external loads into internal stresses
f = flexibility matrix of individual elements
R = column matrix of external loads
w = joint displacements
X = unknowns (forces or joint displacements)
a = transformation matrix of joint displacements into strains
r = stiffness matrix of individual elements
D = $b_1{}^tfb_1$
D_0 = $b_1{}^tfb_0$
C = a^tra
0 = null matrix
t = transpose of a matrix

Introduction

The matrix methods of structural analysis which have appeared recently in the literature are of two classes: the Argyris method and the Klein method. The Argyris method can be subdivided into those in which forces or deformations are taken as unknowns. This is stated clearly in Ref. 1, where the basic references of the two methods are mentioned.

In Ref. 2, Klein exposes the foundations of his method and points out as one of his disadvantages: "Matrix is large." Later, in more recent works,[3,4] Klein advocates a pretriangularization of his initial equations to avoid that disadvantage. He also states that the ideal pretriangularization is obtained when the redundant part of the structural system is isolated. In this case the order of the matrix which has to be inverted is much less than the order of the initial large matrix.

The purpose of this note is to show that Argyris' equations are exactly Klein's after the ideal pretriangularization is obtained. This conclusion allows the unification of all methods of matrix structural analysis.

Theory

By the Argyris formulation, taking forces as unknowns, the internal stresses and the joint displacements of a structure submitted to external loads applied at the joints are, respectively,[5]

$$S = b_0R + b_1X \tag{1}$$

and

$$w = b_0{}^tfS \tag{2}$$

where X is given by

$$DX + D_0R = 0 \tag{3}$$

Equations (1–3) can be written jointly in Table 1, where (b_1) and (b_0) are, respectively, matrices b_1 and b_0 in which the rows corresponding to the independent internal stresses are excluded, and $(b_0{}^tf)$ is matrix $b_0{}^tf$ rearranged so that columns referring to the independent internal stresses are written first.

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Table 1 Pretriangularized equations with forces as unknowns

Internal Stresses S: Independent X	Internal Stresses S: Dependent	Joint displacements W	
D			$-D_0R$
$-(b_1)$	1 1 1 1 1 1		$(b_0)R$
$-(b_0^t f)$		1 1 1 1 1 1	0

Table 2 Pretriangularized equations with displacements as unknowns

Joint displacements X	Internal Stresses S	
C		R
$-ra$	1 1 1 1 1 1 1 1 1 1 1 1	0

If joint displacements are considered as unknowns, the response of the structure to external loads applied at the joints is given in Ref. 5 as

$$CX = R \tag{4}$$

and

$$S = raX \tag{5}$$

Equations (4) and (5) can be written jointly in Table 2.

Argyris' equations, put in the form of Tables 1 and 2, are exactly the equations of Klein when the ideal pretriangularization is attained, a situation in which there exists always a group of equations that are not pretriangularized. This group of equations is constituted in Table 1 by Eq. (3) and in Table 2 by Eq. (4).

Example

Consider the first example from Ref. 2. Taking as redundancies the internal stresses P_3 and P_4, the pretriangularized equations as in Table 1 are, for this case, given in Table 3. Solving the equations from Table 3, the solution given in Ref. 2 is obtained. Incidentally, there is a minor difference: in Ref. 2, with three decimals, $P_3 = 0.226$ and $P_4 = 0.344$. From Table 3, $P_3 = 0.216$ and $P_4 = 0.334$. This difference obviously is due to the fact that in Ref. 2 a matrix of order 16 was inverted and here a matrix of order 2.

Table 3 Pretriangularized equations of the example

P_3	P_4	P_1	P_2	P_5	P_6	q_1a	q_2a	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	
42	-15.33															4
-15.33	16															2
1		1														1
	1		1													1
1				1												
-2	1				1											
1						1										
-1	1						1									
		-2	-1					1								1
		-1	-1						1							
-2	-1									1						
-1	-1										1					
				2								1				
					2								1			
				1										1		
					1										1	

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Equivalence between Chemical-Reaction and Volume-Viscosity Effects in Linearized Nonequilibrium Flows

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In the theory of sound propagation in a relaxing medium, it is a well-established fact that, for wave frequencies much smaller than the relaxation frequency, the relaxation process itself can be considered as having the same effect as a volume viscosity.[1] It is interesting to determine under what conditions the same statement would hold for steady nonequilibrium flows.

It is the purpose of this note to show that this happens for "linearized flows" when the ratio between a macroscopic characteristic time t_M and a suitably defined chemical characteristic time is much greater than one, i.e., near-equilibrium conditions. In these conditions, the basic equations for the linearized motion of a reacting medium are shown to reduce to those pertinent to an equivalent motion, at a Reynolds number defined in terms of appropriate thermodynamic derivatives, of an inert but viscous medium.

Assume the chemical affinity A, the specific volume v, and the entropy per unit mass of the mixture, s, as the basic set of independent thermodynamic variables. The appropriate thermodynamic potential ψ is the first-order Lagrange transform[2, 3] of the specific energy e with respect to the progress variable of the reaction ξ:

$$\psi = e - \xi A = \psi(s,v,A) \tag{1}$$

and the Gibbs relation

$$d\psi = Tds - pdv - \xi dA \tag{2}$$

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